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FLOW AND HEAT TRANSFER OF POWER LAW FLUID FILM ON AN UNSTEADY STRETCHING SURFACE WITH TEMPERATURE DEPENDENT VISCOSITY AND THERMAL CONDUCTIVITY

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Abstract

This paper considers the flow and heat transfer in a non-Newtonian power-law liquid film over a horizontal unsteady stretching sheet. The fluid viscosity and thermal conductivity are assumed to vary with temperature, whereas the Prandtl number is assumed to be constant. The transformed time dependent conservation laws are solved numerically. The velocity, temperature and thickness of the film, heat transfer and the friction at the sheet are found to be significantly affected with the variation of the fluid properties.

1. Introduction

The problem of flow and heat transfer within a thin liquid film on a time dependent stretching sheet is often encountered in many industrial

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applications such as extrusion process, the extruded material issues through a die, manufacturing of plastic and rubber sheets, glass and paper production, the drawing of plastic films and artificial fibers, and foodstuff processing. Following the pioneer work of Crane [12], in which the investigation of the flow over a linearly stretching sheet is carried out. Due to its practical applications, various aspects of the problem have been investigated by many authors [1-5, 11, 14-20]. In all these studies, the fluid surrounding the stretching sheet is infinite and the boundary conditions on the stretching sheet and on the fluid at infinity are prescribed. However, the free-surface flow of liquids (especially non-Newtonian) in thin films is a widely occurring phenomenon in many industrial applications that mentioned above. Accordingly, Wang [21] was the first to study the flow of Newtonian fluid in a thin film over an unsteady stretching sheet. The problem was developed by Andersson et al. [7], who considered the heat transfer. Andersson et al. [6] investigated numerically the hydrodynamic problem of power-law fluid flow within a liquid film over a stretching sheet. Chen [8] made an investigation of the thermal behavior of a power-law fluid film over a flat sheet under unsteady stretching. In addition, the viscous dissipation and Marangoni effects on the flow and heat transfer characteristics within a power-law fluid film on an unsteady stretching sheet have been examined by Chen [9] and [10], respectively. Dandapat et al. [13] studied the effects of variable viscosity, variable thermal conductivity and thermocapillarity on the flow and heat transfer in a laminar Newtonian liquid film on a horizontal stretching sheet. And show in their study how the velocity field, skin friction, temperature distribution and heat transfer changes due to the variation of viscosity, conductivity and thermocapillarity with temperature.

Following the work of Chen [9], this paper is devoted to investigate the effects of variable viscosity and thermal conductivity on the flow and heat transfer occurring in a thin, power-law liquid film over an unsteady stretching sheet.

2. Mathematical Formulation

Consider the flow of a thin, power-law fluid film on a horizontal elastic stretching sheet, as shown in Figure 1. The fluid motion within the film arises due to the stretching of the elastic surface. The continuous sheet aligned with the x-axis at $y = 0$ moves in its own plane with a velocity

$$u_s = \frac{bx}{1 - at}, \tag{1}$$

where a and b are positive constants with dimension $(\text{time})^{-1}$, and the surface temperature of the stretching sheet T_s varies with the horizontal coordinate x and time t as

$$T_s = T_0 - T_{ref}\left(\frac{b^{2-n}x^2}{2\mu_0 / \rho_0}\right)(1 - at)^{n-(5/2)}, \tag{2}$$

where T_0 is the temperature at the slit, and T_{ref} is the reference temperature, such that $0 \leq T_{ref} \leq T_0$.

The fluid viscosity and thermal conductivity are assumed to vary with temperature in the form (see [13]).

$$\mu = \mu_0 e^{-\xi(T - T_0)}, \tag{3}$$

$$k = k_0[1 + c(T - T_0)], \tag{4}$$

where μ_0 and k_0 are the viscosity and thermal conductivity of the fluid, respectively, at slit temperature T_0. For most liquids the viscosity decreases with temperature, i.e., ξ is positive. In general, $c > 0$ for fluids such as water and air, while $c < 0$ for fluids such as lubrication oils.

Under all the above assumptions, the boundary layer form of the governing equations can be written as [8, 9]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_0}\frac{\partial}{\partial y}\left(\mu\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right), \tag{6}$$

$$\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{1}{\rho_0 C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\mu}{C_p}\left|\frac{\partial u}{\partial y}\right|^{n-1}\left(\frac{\partial u}{\partial y}\right)^2. \tag{7}$$

The associated boundary conditions are

$$\text{at } y = 0: \quad u = u_s, \quad \upsilon = 0, \quad T = T_s, \tag{8-a}$$

$$\text{at } y = \ell: \quad \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0, \tag{8-b}$$

$$\text{at } y = \ell: \quad \upsilon = u\frac{\partial \ell}{\partial x} + \frac{\partial \ell}{\partial t}, \tag{8-c}$$

where u and υ are the velocity components along x and y directions, respectively, μ is the viscosity, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the thermal conductivity, T is the temperature of the fluid.

Here, ℓ is the thickness of the liquid film, n is the flow behavior index. The fluid is Newtonian for $n = 1$. The fluids are termed pseudoplastic (or shear thinning) for $n < 1$ and dilatant (or shear thickening) for $n > 1$.

The specific representation of the stretching velocity and surface temperature in Equations (1) and (2), respectively, was chosen to allow the unsteady non-linear partial differential equations (6) and (7) to be converted to a set of ordinary differential equations by means of the similarity transformation.

We introduce the following similarity transformations:

$$\eta = \left(\frac{b^{2-n}}{\mu_0 / \rho_0}\right)^{\frac{1}{n+1}} x^{\frac{1-n}{1+n}}(1 - at)^{\frac{n-2}{n+1}} y, \tag{9}$$

$$\psi = \left(\frac{b^{1-2n}}{\mu_0 / \rho_0}\right)^{\frac{-1}{n+1}} x^{\frac{2n}{n+1}}(1 - at)^{\frac{1-2n}{n+1}} f(\eta), \tag{10}$$

$$T = T_0 - T_{ref}\left(\frac{b^{2-n}x^2}{2\mu_0/\rho_0}\right)(1-at)^{n-(5/2)}\theta(\eta), \tag{11}$$

where f is a dimensionless stream function, η is a similarity space variable and θ is the dimensionless temperature. In Equation (10), the stream function $\psi(x, y)$ is defined by $u = \frac{\partial\psi}{\partial y}$ and $\upsilon = -\frac{\partial\psi}{\partial x}$, such that the continuity Equation (5) is satisfied automatically.

By using the transformations (8)-(11), the governing Equations (6)-(7) and the boundary conditions (8) become

$$\left(e^{A\theta}|f''|^{n-1}f''\right)' + \frac{2n}{n+1}ff'' - (f')^2 - S\left(f' + \left(\frac{2-n}{1+n}\right)\eta f''\right) = 0, \tag{12}$$

$$\frac{1}{\Pr}((1-\delta\theta)\theta')' + \left(\frac{n-2}{n+1}\eta S + \frac{2n}{n+1}f\right)\theta' -$$

$$\left(2f' - S\left(n - \frac{5}{2}\right)\right)\theta + \mathrm{Ec}\, e^{A\theta}|f''|^{n-1}(f'')^2 = 0. \tag{13}$$

With the boundary conditions

$$f'(0) = 1, \quad f(0) = 0, \quad \theta(0) = 1, \tag{14-a}$$

$$f(\beta) = S\beta\left(\frac{2-n}{2n}\right), \tag{14-b}$$

$$f''(\beta) = 0, \quad \theta'(\beta) = 0, \tag{14-c}$$

where $S = a/b$ is the unsteadiness parameter, β denotes the value of η at the free surface, $A = -\xi(T_s - T_0)$ is the viscosity variation parameter, $\delta = -c(T_s - T_0)$ is the thermal conductivity variation parameter, $\Pr = (\rho_0 x u_s/\mu_0)\mathrm{Re}_x^{\frac{-2}{n+1}}$ is the generalized Prandtl number, $Re_x = \rho_0 x^n u_s^{2-n}/\mu_0$ is the local Reynolds number, $\mathrm{Ec} = u_s^2/c_p$ $(T_s - T_0)$ is the Eckert number and the Primes designate differentiation with respect to η only. It is worthy mention herein that, the dimensionless film thickness β is unknown constant which should be determined as an integral part of the boundary value problem.

The physical quantities of practical importance in this problem are the velocity components u and υ, the local skin friction coefficient C_f and the local Nusselt number Nu_x which are defined as:

$$u = u_s f'(\eta), \tag{15}$$

$$\upsilon = -u_s \mathrm{Re}_x^{\frac{-1}{n+1}}\left(\frac{2n}{n+1} f + \frac{1-n}{1+n}\eta f'\right), \tag{16}$$

$$C_f = 2e^A \mathrm{Re}_x^{\frac{-1}{n+1}}[-f''(0)]^n, \tag{17}$$

$$\mathrm{Nu}_x = \frac{1}{2}(1-at)^{-\frac{1}{2}}\mathrm{Re}_x^{\frac{n+2}{n+1}}\theta'(0). \tag{18}$$

3. Results and Discussion

The system of nonlinear Equations (12) and (13) subject to the boundary conditions (14) is solved numerically. Equations (12) and (13) were first formulated as a set of five first-order equations. For a tentative value of β, this set subjected to the three explicit initial conditions in (14-a), the explicit terminal conditions (14-c) was solved by the fourth order Runge-Kutta integration scheme with a modified version of the Newton-Raphson shooting method. The numerical solution did generally not satisfy the auxiliary terminal condition (14-b), and the estimated value of β, was therefore systematically adjusted until Equation (14-b) was satisfied to within 10^{-6}. To assess the validity and accuracy of the present numerical scheme, some skin friction results have been compared to those of Chen [9]. This comparison shows excellent agreement between the results, which are presented in Table 1.

The results of the numerical computations are displayed in Figures 2-10, which illustrate the effect of various physical parameters on the flow and heat transfer aspects of the present study. The parametric study is focused on the effects of the viscosity and thermal conductivity variation parameters.

For the hydrodynamic problem, the critical values of the steadiness parameter $S0,$ above which no solution could be obtained, for the Newtonian fluid film $(n = 1)$ and non-Newtonian fluid film $(n = 0.8, 1.2)$ are found to be agree with those of Chen [8]. Figure 2 shows the variation of the dimensionless film thickness β with the steadiness parameter S. Figure shows that the values $S0$ are about 1.35 for $n = 0.8$ (shear-thinning fluid), 2.0 for $n = 1.0$ (Newtonian fluid) and 3.0 for $n = 1.2$ (shear-thickening fluid). The figure reveals that at fixed value of S the dimensionless film thickness β increases as n increases. Also, It is obvious from Figure 2 that the film thickness increases with increasing M for shear thinning, Newtonian and shear thickening liquid films and the effect of the viscosity and thermal conductivity variation on β vanishes when $S \to S_0$ (the liquid film thickness becomes infinitesimal).

The effect of the viscosity and thermal conductivity variation on the film thickness, the velocity and temperature of the free surface are shown in Figures 3, 4 and 5, respectively. Figure 3 indicates that the film thickness β increases as A increases for the Newtonian and the power-law fluids. Further, it is to be noted that the free surface velocity $f'(\beta)$ is decreasing with increasing A due to the thickening of the liquid layer for all values of n as depicted in Figure 4. From Figure 5, it can be noted that the free surface temperature $\theta(\beta)$ of pseudo-plastic fluid film is more sensitive to the viscosity and thermal conductivity variation than that of Newtonian and dilatant fluid films. In addition, it is found that $\theta(\beta)$ decreases as A increases when $n = 0.8, 1.0$ while the effect of A on $\theta(\beta)$ is insensible at $n = 1.2$.

Velocity and temperature distributions for $S = 0.8, \Pr = 3.0$ and $\mathrm{Ec} = 0.2$ are presented in Figures 6, 7 and 8 for pseudo-plastic $(n = 0.8)$, Newtonian $(n = 1.0)$ and dilatant $(n = 1.2)$ fluids, respectively, at selected values of A and δ. From these figures, it is clear that for all considered values of n the film thickness β is seen to be increased as a result of the viscosity variation effect. In addition, it can be seen that near the stretching sheet the velocity increases as the viscosity increases. Also the figures shows that the dimensionless temperature decreases monotonously with the η for all values of the governing parameters. This behavior is simply reflecting the gradual increase in actual temperature T from T_s at $y = 0$ to the free-surface temperature. Moreover, the dimensionless temperature decreases as A increases while it increases, when δ changes from 0 to -0.1.

Variations of local skin friction coefficient C_f and the local Nusselt number, in terms of the sheet shear stress $-f''(0)$ and the wall temperature gradient $-\theta'(0)$, are presented in Figures 9 and 10 for various values of A and n. These figures illustrate that C_f decreases while Nu_x increases with the increase in viscosity variation A for Newtonian and non-Newtonian fluid film. In addition, for Newtonian and non-Newtonian heat flow from the liquid at the stretching sheet is reduced when conductivity changes from 0 to -0.1 as shown in Figure 10.

Finally, Figures 11 and 12 show how the flow field affected by considering the variation of the fluid properties as a function of temperature. For shear thinning fluid (as an example), in the case of constant viscosity, it is obvious that the momentum Equation (12) is uncoupled from the energy Equation (13). Thereby, changes in the values of Pr and Ec cause no change in the velocity profile f' as illustrated by the soled curves. Therefore, considering the temperature-dependent viscosity is important and it makes the parameters related to the energy equation have a marked effect on the flow field.

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Nomenclature

A	viscosity variation parameters	$x,\ y$	Cartesian coordinates
a	stretching rate	**Greek symbols**	
C_f	local friction coefficient	β	dimensionless film thickness
c_p	specific heat at constant pressure	δ	thermal conductivity variation parameter
Ec	Eckert number	η	similarity space variable
f	dimensionless stream function	ξ	constant in Equation (3)
k	thermal conductivity	μ	viscosity
ℓ	film thickness	θ	dimensionless temperature
n	power-law index	ρ	fluid density
Nu_x	local Nusselt number	ψ	stream function
Pr	generalized Prandtl number	**Subscripts**	
Re_x	the Reynolds number	0	origin
S	unsteadiness parameter	ref	reference value
t	time	s	sheet
T	temperature	x	local value
u	velocity component the x-direction	**Superscripts**	
υ	velocity component the y-direction		differentiation with respect to η

Table 1. Comparison of $f''(0)$ for $\mathrm{Pr} = 10$, $A = 0.0$, $\delta = 0.0$, and for various values of n and S parameters to Chen [9]

n	S	Chen [9]	Present results
0.8	0.8	-1.22301	-1.22306
	1.2	-0.77919	-0.77925
1.0	0.8	-1.24581	-1.24581
	1.2	-1.27918	-1.27917
1.2	0.8	-1.21778	-1.21778
	1.2	-1.30883	-1.30883

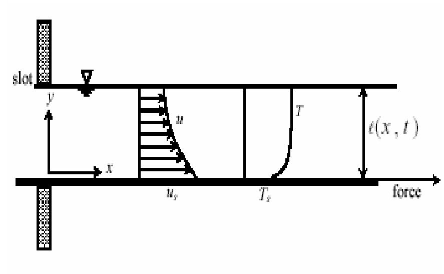

Figure 1. Schematic representation of a liquid film on an unsteady stretching sheet.

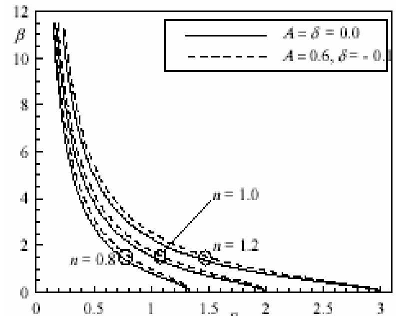

Figure 2. Variation of film thickness β against unsteadiness parameter S at $\mathrm{Pr} = 3.0, \mathrm{Ec} = 0.2$.

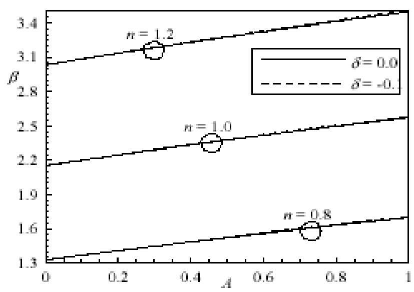

Figure 3. Variation of film thickness β against viscosity variation parameter A at $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

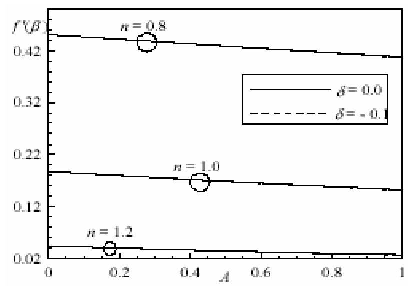

Figure 4. Variation of free surface velocity $f'(\beta)$ against viscosity variation parameter A at $\mathrm{Pr} = 3.0, \mathrm{Ec} = 0.2$ and $S = 0.8$.

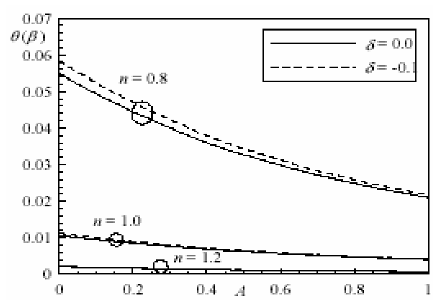

Figure 5. Variation of free surface Temperature θ(β) against viscosity variation parameter A at $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

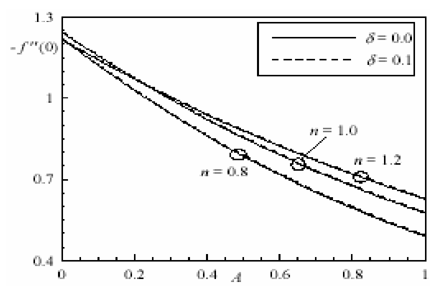

Figure 6. Variation of local skin friction coefficient in terms of $-f''(0)$ against viscosity variation parameter A at $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

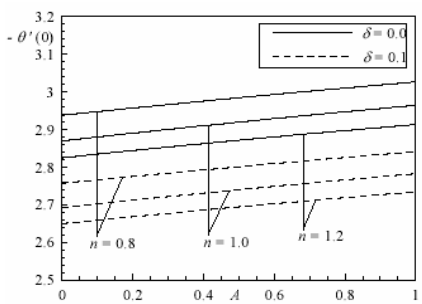

Figure 7. Variation of the local Nusselt number in terms of $-\theta'(0)$ against viscosity variation parameter A at $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

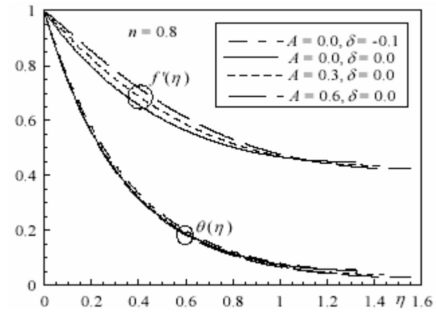

Figure 8. Velocity and temperature distribution for $n = 0.8$, $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

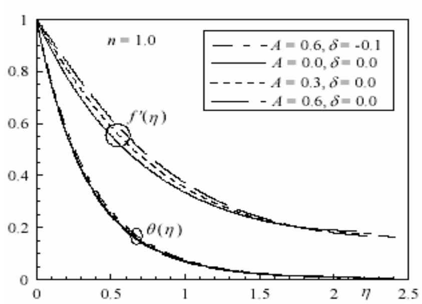

Figure 9. Velocity and temperature distribution for $n = 1.0$, $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

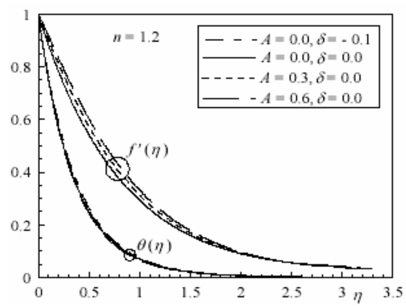

Figure 10. Velocity and temperature distribution for $n = 1.2$, $\mathrm{Pr} = 3.0$, $\mathrm{Ec} = 0.2$ and $S = 0.8$.

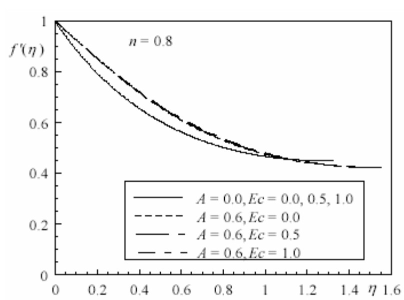

Figure 11. Velocity distribution for $n = 0.8, \mathrm{Pr} = 3.0, \delta = 0.0$ and $S = 0.8$.

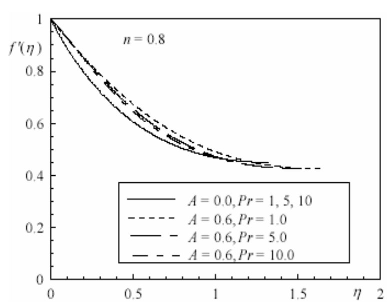

Figure 12. Velocity distribution for $n = 0.8, \mathrm{Ec} = 0.2, \delta = 0.0$ and $S = 0.8$.

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